Conclusions

New data have been obtained for the flow issuing from a round-corner sharp-edged rectangular orifice. When compared to the previously published data from a similar orifice, but with sharp corners, these new data show that the round corners have no discernible effect upon either the magnitude or location of the saddle-back velocity profile.

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Harmonic Temperature Effect on Vibrations of an Orthotropic Plate of Varying Thickness

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Introduction

THE interest in the effect of temperature on solid bodies has increased as a result of recent developments in space technology, high-speed atmospheric flights, and nuclear energy applications. Notable contributons are available on the vibrations of orthotropic rectangular plates.[1-4] It is well known[5] that, in the presence of a constant thermal gradient, the elastic coefficients of homogeneous materials become functions of the space variable. Fanconneau and Marangoni[6] have investigated the effect of the nonhomogeneity caused by a thermal gradient on the natural frequencies of a simply supported plate of uniform thickness. Recently, Tomar and Gupta[7,8] have considered the effect of a thermal gradient on the frequencies of an orthotropic plate of variable thickness. The object of this study is to determine the effect of the harmonic temperature distribution on the frequencies of an orthotropic rectangular plate of linearly varying thickness. Here, the quintic spline technique has been used to compute the frequencies for the first two modes of vibration for different combinations of boundary conditions and for various values of aspect ratio, taper constant, and temperature constant.

Analysis and Equation of Motion

It is assumed that the rectangular orthotropic material is subjected to an harmonic temperature distribution along its length, i.e., in the x direction

$$T = T_0 \cos(\pi/2)x \tag{1}$$

where T denotes the temperature excess above the reference temperature at any point at a distance $X = x/a$ and T_0 the temperature excess above the reference temperature at the end $x = a$ or $X = 1$.

The temperature dependence of the modulus of elasticity for most orthotropic materials is given by

$$E_x(T) = E_1(1-\gamma T),\ E_y(T) = E_2(1-\gamma T),\ G_{xy}(T) = G_0(1-\gamma T) \tag{2}$$

where E_1 and E_2 are the values of Young's moduli in the x and y directions, respectively, and G_0 the value of the shear modulus between directions x and y at the reference temperature, i.e., at $T = 0$.

Taking as a reference the temperature at the end of the plate, i.e., at $X = 1$, the modulus variations in view of Eqs. (1) and (2) become

$$E_x(X) = E_1[1-\alpha\cos(\pi/2)X],\ E_y(X) = E_2[1-\alpha\cos(\pi/2)X]$$

$$G_{xy}(X) = G_0[1-\alpha\cos(\pi/2)X] \tag{3}$$

where $\alpha = \gamma T_0 (0 \le \alpha < 1)$, a parameter representing the temperature constants.

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The governing differential equation of transverse motion of an orthotropic rectangular plate of variable thickness is[7]

$$D_x w_{,xxxx} + D_y w_{,yyyy} + 2H w_{,xxyy} + 2H_{,x} w_{,xyy} + 2H_{,y} w_{,xxy} + 2D_{x,x} w_{,xxx} + 2D_{y,y} w_{,yyy} + D_{x,xx} w_{,xx} + D_{y,yy} w_{,yy} + D_{1,xx} w_{,yy} + D_{1,yy} w_{,xx} + 4D_{xy,xy} w_{,xy} + \rho h w_{,tt} = 0 \tag{4}$$

where

$$D_x = E_x h^3/12(1-\nu_x\nu_y), \quad D_y = E_y h^3/12(1-\nu_x\nu_y),$$

$$D_{xy} = G_{xy}h^3/12, \quad D_1 = \nu_x D_y (=\nu_y D_x), \quad H = D_1 + 2D_{xy}$$

and where ν_x and ν_y are the Poisson's ratio in x and y directions, respectively, w the transverse deflection, ρ the mass density per unit volume, t the time, h the thickness, a the length, and b the width. A comma followed by a suffix denotes partial differentiation with respect to that variable.

Since the thickness varies in the x direction only, one may find that h, D_x, D_y, and D_{xy} of the plate become functions of x only. Let the two opposite edges $y=0$ and $y=b$ of the plate be simply supported, so that the plate undergoing free transverse vibrations with frequency p may have a levy solution as

$$w(x,y,t) = \bar{W}(x)\sin(\pi y/b)e^{ipt} \tag{5}$$

Introducing the nondimensional variables

$$\bar{H} = \frac{h}{a}, \quad W = \frac{\bar{W}}{a}, \quad X = \frac{x}{a}, \quad D_X = \frac{D_x}{a^3}, \quad D_Y = \frac{D_y}{a^3}, \quad D_{XY} = \frac{D_{xy}}{a^3} \tag{6}$$

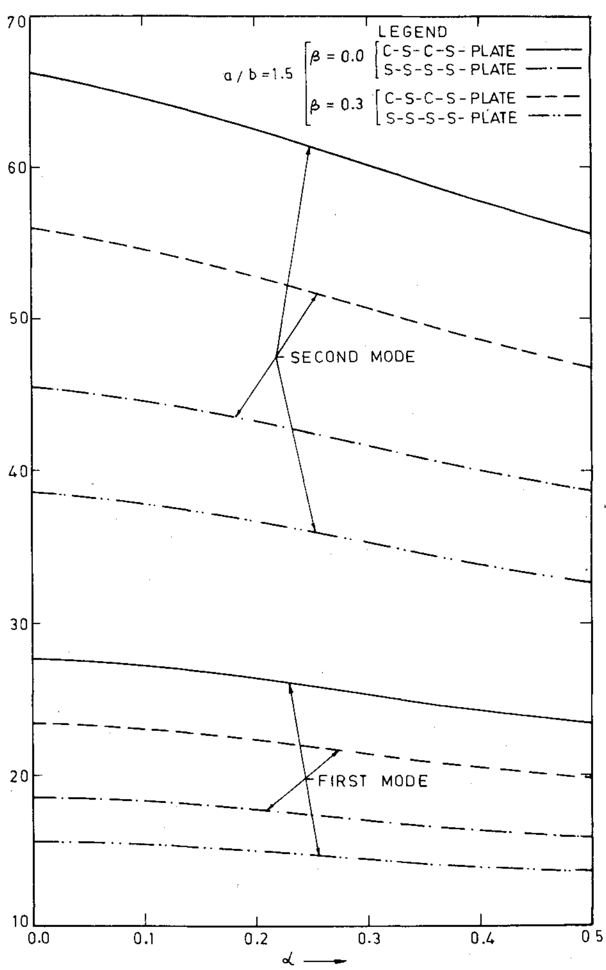

Fig. 1 Variation of frequency with temperature constant.

one gets the expression for rigidities,

$$D_X = E_1[1-\alpha\cos(\pi/2)X]\bar{H}^3/12(1-\nu_x\nu_y)$$

$$D_Y = E_2[1-\alpha\cos(\pi/2)X]\bar{H}^3/12(1-\nu_x\nu_y)$$

$$D_{XY} = G_0[1-\alpha\cos(\pi/2)X]\bar{H}^3/12 \tag{7}$$

Assuming the thickness variation of the plate as

$$\bar{H}(X) = H_0(1-\beta X) \tag{8}$$

where β is the taper constant and $H_0 = \bar{H}|_{X=0}$. Substituting Eqs. (5-8) into Eq. (4), one obtains the equation of motion as

$$\left[1-\alpha\cos\frac{\pi}{2}X\right](1-\beta X)^2 W_{,xxxx} + 2\left[\alpha\frac{\pi}{2}\sin\frac{\pi}{2}X \times (1-\beta X)^2 - 3\beta\left(1-\alpha\cos\frac{\pi}{2}X\right)(1-\beta X)\right]W_{,XXX}$$
$$+\left[\alpha\left(\frac{\pi}{2}\right)^2\cos\frac{\pi}{2}X(1-\beta X)^2 - 6\alpha\beta\frac{\pi}{2}\cdot\sin\frac{\pi}{2}X(1-\beta X) + 6\beta^2\left(1-\alpha\cos\frac{\pi}{2}X\right) - 2r^2\left(\frac{E^*+2G_0}{E_1^*}\right) \times\left(1-\alpha\cos\frac{\pi}{2}X\right)(1-\beta X)^2\right]W_{,XX} + \left\{-2r^2\left(\frac{E^*+2G_0}{E_1^*}\right)\right.$$
$$\left.\times\left[\frac{\alpha\pi}{2}\cdot\sin\frac{\pi}{2}X(1-\beta X)^2 - 3\beta\left(1-\alpha\cos\frac{\pi}{2}X\right)\times(1-\beta X)\right]\right\}W_{,X} + \left\{r^4\frac{E_2^*}{E_1^*}\left(1-\alpha\cos\frac{\pi}{2}X\right)(1-\beta X)^2\right.$$
$$\left.-r^2\frac{E^*}{E_1^*}\left[\alpha\cdot\left(\frac{\pi}{2}\right)^2\cos\frac{\pi}{2}X(1-\beta X)^2 - 6\alpha\beta\frac{\pi}{2}\times\sin\frac{\pi}{2}X(1-\beta X) + 6\beta^2\left(1-\alpha\cos\frac{\pi}{2}X\right)\right]\right\}W = \lambda^2 W \tag{9}$$

where the frequency parameter is

$$\lambda^2 = \frac{p^2a^2}{E_1^*/\rho}\cdot\frac{12}{H_0^2} \tag{10}$$

and

$$r^2 = (\pi a/b)^2, \qquad E_1^* = [E_1/(1-\nu_x\nu_y)]$$

$$E_2^* = [E_2/(1-\nu_x\nu_y)], \qquad H^* = \nu_{xy}E_1^*/(=\nu_x E_2^*)$$

Solution

Choose $(n+1)$ points $X_0, X_1, X_2, \ldots, X_n$ in the range $0 \le X \le 1$ such that $0 = X_0 < X_1 < X_2 < \ldots < X_n = 1$.

Assuming the solution for $W(X)$ in quintic spline form as

$$W(X) = a_0 + \sum_{j=1}^{4} a_j(X-Xo)^j + \sum_{i=0}^{n-1} b_i(X-X_i)_+^5 \tag{11}$$

where

$$(X-X_i)_+ = 0 \quad \text{if } X < X_i$$
$$= X - X_i \quad \text{if } X \ge X_i \tag{12}$$

It is also assumed, for simplicity, that the knots X_i are equally spaced in (0,1) with the spacing interval ΔX, so that

$$\Delta X = 1/n, \quad X_i = i\Delta X (i=0,1,2,\ldots,n) \tag{13}$$

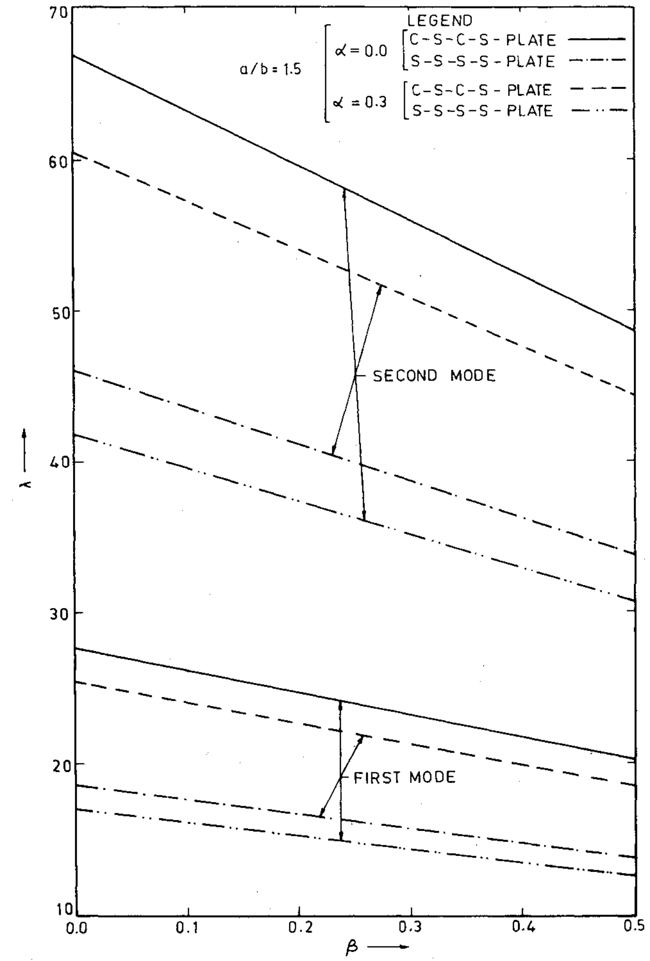

Fig. 2 Variation of frequency with taper constant.

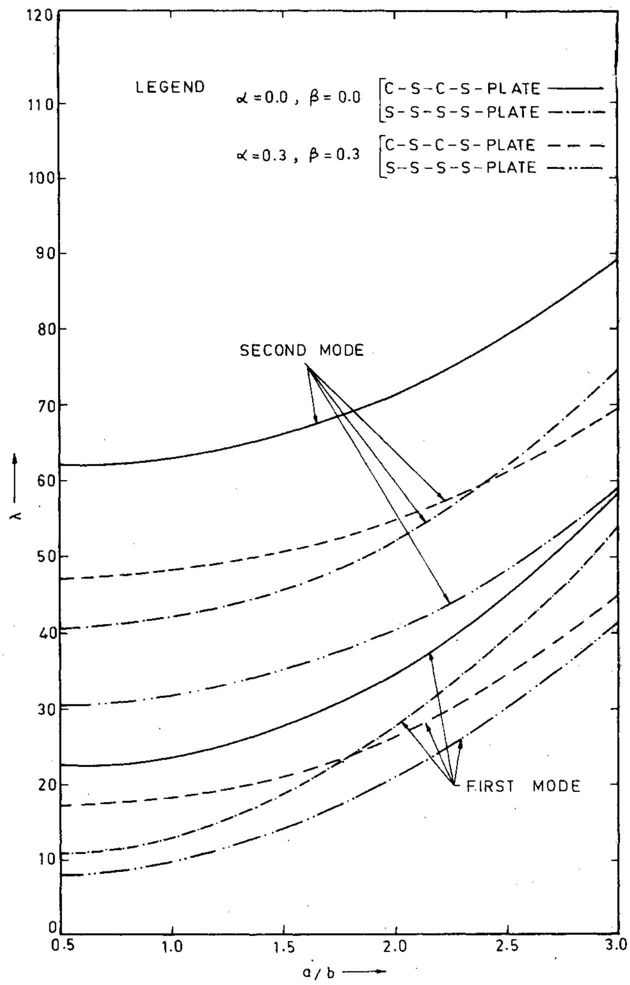

Fig. 3 Variation of frequency with aspect ratio.

Substituting $W(X)$ from Eq. (11) into Eq. (9), gives, for satisfaction at the mth knot,

$$B_4 a_0 [B_4(X_m - X_0) + B_3] a_1 + [B_4(X_m - X_0)^2 + 2B_3(X_m - X_0) + 2B_2] a_2 + [B_4(X_m - X_0)^3 + 3B_3(X_m - X_0)^2 + 6B_2(X_m - X_0) + 6B_1] a_3 + [B_4(X_m - X_0)^4 + 4B_3(X_m - X_0)^3 + 12B_2(X_m - X_0)^2 + 24B_1(X_m - X_0) + 24B_0] a_4 + \sum_{i=0}^{n-1} [B_4(X_m - X_i)^5 + 5B_3(X_m - X_i)^4 + 20B_2(X_m - X_i)^3 + 60B_1(X_m - X_i)^2 + 120B_0(X_m - X_i)] b_i = 0 \qquad (14)$$

where

$$B_0 = \left(1 - \alpha\cos\frac{\pi}{2}X_m\right)\cdot(1 - \beta X_m)^2$$

$$B_1 = 2\left[\alpha\frac{\pi}{2}\sin\frac{\pi}{2}X_m\cdot(1 - \beta X_m)^2 - 3\beta\left(1 - \alpha\cos\frac{\pi}{2}X_m\right)(1 - \beta X_m)\right]$$

$$B_2 = \left[\alpha\left(\frac{\pi}{2}\right)^2\cos\frac{\pi}{2}X_m\cdot(1 - \beta X_m)^2 - 6\alpha\beta\frac{\pi}{2}\cdot\sin\frac{\pi}{2}X_m\cdot(1 - \beta X_m) + 6\beta^2\left(1 - \alpha\cos\frac{\pi}{2}\right) \ldots - 2r^2\left(\frac{E^* + 2G_0}{E_I^*}\right)\left(1 - \alpha\cos\frac{\pi}{2}X_m\right)\cdot(1 - \beta X_m)^2\right]$$

$$B_3 = -2r^2\left(\frac{E^* + 2G_0}{E_I^*}\right)\left[\alpha\cdot\frac{\pi}{2}\cdot\sin\frac{\pi}{2}X_m \times (1 - \beta X_m)^2 - 3\beta\left(1 - \alpha\cos\frac{\pi}{2}X_m\right)(1 - \beta X_m)\right]$$

$$B_4 = \left\{r^4\frac{E_2^*}{E_I^*}\left(1 - \alpha\cos\frac{\pi}{2}X_m\right)(1 - \beta X_m)^2 - r^2\frac{E^*}{E_I^*}\left[\alpha\cdot\frac{\pi}{2}\cos\frac{\pi}{2}X_m\cdot(1 - \beta X_m)^2 - 6\alpha\beta\cdot\frac{\pi}{2}\cdot\sin\frac{\pi}{2}X_m\cdot(1 - \beta X_m) + 6\beta^2\left(1 - \alpha\cos\frac{\pi}{2}X_m\right)\right] - \lambda^2\right\} \qquad (15)$$

Thus, one obtains a homogeneous set of equations in terms of unknown constants $a_0, a_1, a_2, a_3, a_4, b_0, b_1, \ldots, b_{n-1}$, which, when written in matrix notation, takes the form

$$[B][C] = [0] \qquad (16)$$

where $[B]$ is an $(n+1)\times(n+5)$ matrix and $[C]$ an $(n+5)\times 1$ matrix.

Boundary Conditions and Frequency Equations

The frequency equations have been obtained by employing the boundary conditions for clamped (C) and simply supported (S) plates.

C-S-C-S Plates

Applying the boundary conditions of Eq. (11) for clamped edges $X=0$ and $X=1$, one gets

$$[A_1][C]=[0] \tag{17}$$

where $[A_1]$ is a $4\times(n+5)$ and $[C]$ an $(n+5)\times 1$ matrix. Thus, the frequency equation for C-S-C-S plates is

$$|B/A_1|=0 \tag{18}$$

S-S-S-S Plates

Employing the boundary conditions of Eq. (11) for simply supported edges $X=0$ and $X=1$, one gets the boundary equations for S-S-S-S plate as

$$[A_2][C]=0 \tag{19}$$

where $[A_2]$ is a $4\times(n+5)$ and $[C]$ an $(n+5)\times 1$ matrix.

Hence, the frequency equation for an S-S-S-S plate is

$$|B/A_2|=0 \tag{20}$$

Results and Discussion

Frequency equations (18) and (20) are transcendental in λ^2 from which an infinite number of roots can be determined. The frequencies λ corresponding to the first two modes of vibration of C-S-C-S and S-S-S-S orthotropic rectangular plates have been computed for various values of a/b, α, and β. The orthotropic material parameters are taken as[4]

$$E_2^*/E_1^*=0.32,\ E^*/E_1^*=0.04,\ G_0/E_1^*=0.09$$

It is concluded from Figs. 1 and 2 that the frequencies in the first two modes of vibration decrease with the increase in the temperature constant α or taper constant β for both boundary conditions. Further, from Fig. 3, one observes that the frequencies in the first two modes of vibration increase with the increase in aspect ratio a/b for both boundary conditions. It can also be observed from Figs. 1-3 that frequencies in the C-S-C-S plate are higher than in the S-S-S-S plate. λ^2 for $\alpha=\beta=0$ and $a/b=0.5$ and 1.0 is also calculated for the S-S-S-S plate and compared with Dickinson[3] and Hearmon.[4] The values obtained are in close agreement.

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A Simple and Accurate Expression for the Viscosity of Nonpolar Diatomic Gases up to 10,000 K

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Introduction and Theoretical Background

FOR any analytical or numerical solution of a viscous flowfield, an expression for the fluid viscosity is required. The viscosity normally enters the analysis through the Reynolds number. In turn, this number characterizes the type of flow (laminar or turbulent). The Reynolds number is an essential parameter in estimating the viscous forces when a solid body is immersed in the fluid. The Reynolds number is needed for evaluating the drag coefficient C_D of the solid body and the Nusselt number N_u. Both of these parameters are necessary for energy and momentum transfer calculations.

Therefore, it is advantageous to express the fluid viscosity as a function of temperature. The simpler this expression can be, the more convenient its use can be in any analytical or numerical solution of a given flowfield.

A variety of theoretically derived expressions for the gas viscosity can be found in Ref. 1. Unfortunately, most of them are too complicated to be readily used in either analytical or numerical calculations. Probably, the most common expression is the one suggested by van Driest.[2] The derivation of this expression is based on Sutherland's model for viscosity[1] and is

$$\frac{\mu}{\mu_0}=\frac{T_0+S}{T+S}\left(\frac{T}{T_0}\right)^{1.5} \tag{1a}$$

where μ and T are the dynamic viscosity and absolute temperature, respectively, S the so-called Sutherland constant, and μ_0 and T_0 the reference values of the dynamic viscosity and absolute temperature.

Equation (1) can also be written in an alternative way as

$$\mu=a_0T^{1.5}/(T+S) \tag{1b}$$

where $a_0=\mu_0(T_0+S)T_0^{-1.5}$. This equation was found to agree with experimental data in the temperature range from 78 K up to about 1200 K. In order to extend this range Gottlieb and Ritzel[3] modified Eq. (1b). They suggested the following relation between μ and T:

$$\mu=\frac{a_0T^{1.5}}{T+S}\left[1+1.53\times10^{-4}\left(\frac{T}{S}-1\right)^2\right] \tag{2}$$

The addition of the term appearing in the square brackets of Eq. (2) increased the upper temperature limit to 2400 K.

Another familiar and frequently used correlation between the viscosity and temperature is

$$\mu/\mu_0=(T/T_0)^\omega \tag{3}$$

where ω is a number within the range $0.5\leq\omega\leq1$. The value $\omega=0.76$ is probably the most frequently used,[4,5]

$$\mu/\mu_0=(T/T_0)^{0.76} \tag{4}$$

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